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## PEDESTRIAN EXCITATION OF BRIDGES - RECENT RESULTS

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### Abstract

The unexpected pedestrian-excited vibration of London's Millennium Bridge was caused by its low damping and high live load. Several other examples of bridges which vibrate significantly when carrying a crowd of people have since come to light.

This paper reviews results for the dynamic loading caused by moving pedestrians, for both vertical and lateral vibration. The phenomenon of "synchronisation" by which people respond naturally to an oscillating bridge when this has a frequency close to their natural walking or running frequency is an important factor in increasing the severity of loading.

By increasing modal damping, synchronisation can be prevented. This is how the London Millennium Bridge's problem was solved. But how much damping is needed in any particular situation? By making some simplifying assumptions about how people walk or run, it is possible to predict minimum required damping levels to ensure that synchronisation does not lead to high vibration levels. These predictions are compared with published bridge response data and found to be in reasonable agreement.

## INTRODUCTION

To mark the Millennium, a new footbridge was built across the river Thames in London. When the bridge was opened, it was found to sway noticeably. With a large number of pedestrians, its sideways movement was sufficient to cause people to stop walking and hold on to the hand-rails. Video pictures showed that lateral movements up to 75 mm amplitude occurred with frequencies in the range 0.8 to 1 Hz. Probably higher amplitudes occurred periodically and several modes were involved. The deck movement was sufficient to be uncomfortable and to raise concern for public safety. It took 18 months to research the problem and make the necessary modifications.

## BACKGROUND

A report in 1972 quoted by Bachmann and Ammann in their IABSE book (1987), described how a new steel footbridge had experienced strong lateral vibration during an opening ceremony with 300-400 people. They explained how the lateral sway of a person's centre of gravity occurs at half the walking pace. Since the footbridge had a lowest lateral mode of about 1.1 Hz, and people typically walk at about 2 paces/second, their frequency of excitation is 1 Hz which is close to this natural frequency. Thus *"an almost resonating vibration occurred. Moreover it could be supposed that in this case the pedestrians synchronised their step with the bridge vibration, thereby enhancing the vibration considerably"* (Bachmann, 1992, p. 636). The problem was solved by the installation of horizontal tuned vibration absorbers.

A later paper by Fujino et al. (1993) described observations of pedestrian-induced lateral vibration of a cable-stayed steel box girder bridge of similar size to the Millennium Bridge. It was found that when a large number of people were crossing the bridge (2,000 people on the bridge), lateral vibration of the bridge deck at 0.9 Hz could build up to an amplitude of 10 mm, while some of the supporting cables whose natural frequencies were close to 0.9 Hz vibrated with an amplitude of up to 300 mm. By analysing video recordings of pedestrians' head movement, Fujino concluded that lateral deck movement encourages pedestrians to walk in step and that synchronisation increases the human force and makes it resonate with the bridge deck. He summarised his findings as follows: *"The growth process of the lateral vibration of the girder under the congested pedestrians can be explained as follows. First a small lateral motion is induced by the random lateral human walking forces, and walking of some pedestrians is synchronised to the girder motion. Then resonant force acts on the girder, consequently the girder motion is increased. Walking of more pedestrians are synchronised, increasing the lateral girder motion. In this sense, this vibration was a self-excited nature. Of course, because of adaptive nature of human being, the girder amplitude will not go to infinity and will reach a steady state."*

Enquiries subsequent to the opening of the London Millennium Bridge identified some other interesting examples of pedestrian-excited bridge vibration (see

Dallard et al. 2001), including the surprising vibration of the Auckland Harbour Bridge in New Zealand. This is an 8-lane motorway bridge, with three separate parallel roadways. In 1975, one roadway, with two traffic lanes, was closed to vehicles to allow a large crowd of walkers to pass over the bridge. Contemporary newsreel footage shows the crowd walking in step as the roadway built up a large amplitude lateral vibration at about 0.6 Hz. This vibration was serious enough for stewards to go through the crowd calling for walkers to break step, when it subsided naturally. It is interesting that the walkers had not intended to march in step, but had naturally fallen into step with each other, apparently after the bridge began to sway.

## PEDESTRIAN LOADING DATA

The book by Bachmann and Ammann (1987) discusses loading from human motions, distinguishing between walking, running, skipping and dancing. For walking and running, the authors point out that dynamic pavement load is dominated by the pacing frequency (table 1).

Published data on dynamic loads is thin, but Bachmann and Ammann quote an example for a pedestrian walking at 2Hz when the fundamental component (at 2 Hz) of vertical dynamic loading is 37% of static weight and the fundamental component (at 1 Hz) of lateral dynamic loading is 4% of static weight. In the vertical case, harmonics are less than about 30% of the fundamental in amplitude (a typical load-time history is shown in fig. 1); in the lateral case there may be a significant 3<sup>rd</sup> harmonic and an example is quoted in which the 3<sup>rd</sup> harmonic exceeds the lateral fundamental in amplitude.

	Pacing frequency	Forward speed	Stride length	Vertical fundamental frequency	Horizontal fundamental frequency
	$f$ (Hz)	$V$ (m/s)	$L$ (m)	$F_{vert}$ (Hz)	$F_{lat}$ (Hz)
Slow walk	1.7	1.1	0.60	1.7	0.85
Normal walk	2.0	1.5	0.75	2.0	1.0
Fast walk	2.3	2.2	1.00	2.3	1.15
Slow running (jogging)	2.5	3.3	1.30	2.5	1.25
Fast running (sprinting)	>3.2	5.5	1.75	>3.2	>1.6

Table 1: Data on walking and running from Bachmann and Ammann (1987).

These forces are for people walking on stationary pavements, but it is noted by Bachmann and Ammann that *“pedestrians walking initially with individual pace on a footbridge will try to adjust their step subconsciously to any vibration of the pavement. This phenomenon of feedback and synchronisation becomes more pronounced with larger vibration of the structure.”* Also, for vertical vibration, the authors note that displacements of the order of 10-20 mm have to occur for the

phenomenon to be noticeable, although they say that it is more pronounced for lateral vibrations. *“Presumably, the pedestrian, having noticed the lateral sway, attempts to reestablish his balance by moving his body in the opposite direction; the load he thereby exerts on the pavement, however, is directed so as to enhance the structural vibration.”*

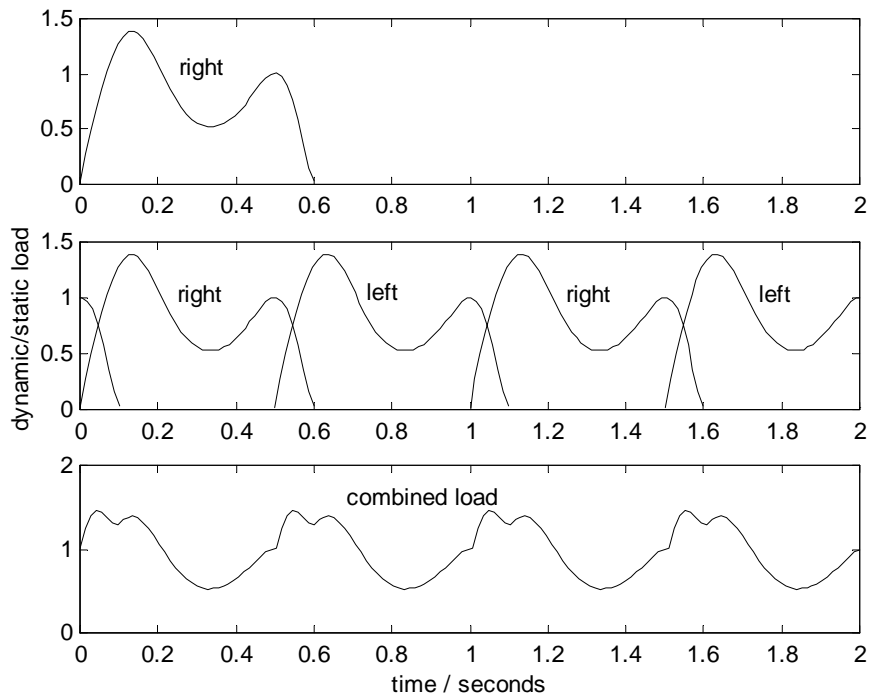


Fig. 1 Vertical load versus time graph from footfall during walking at 2 paces/second (after Bachmann and Ammann, 1987).

## SYNCHRONISATION

Fujino et al. (1993) estimated from video recordings of crowd movement that some 20% or more of pedestrians on their bridge were walking in synchronism with the bridge's lateral vibration which had a frequency of about 0.9 Hz and an amplitude of about 10 mm. The authors computed the amplitude of steady-state lateral vibration of this bridge, first using Bachmann and Ammann's figure of 23 N for the amplitude of lateral force per person and assuming that the pedestrians walk with random phase, and secondly using a force per person of 35 N and the measured result that 20% of them were synchronised to bridge movement. For the random phase case, the calculated amplitude was about 1 mm response; for the 20% correlated case it was about 15 mm (compared with 10 mm measured).

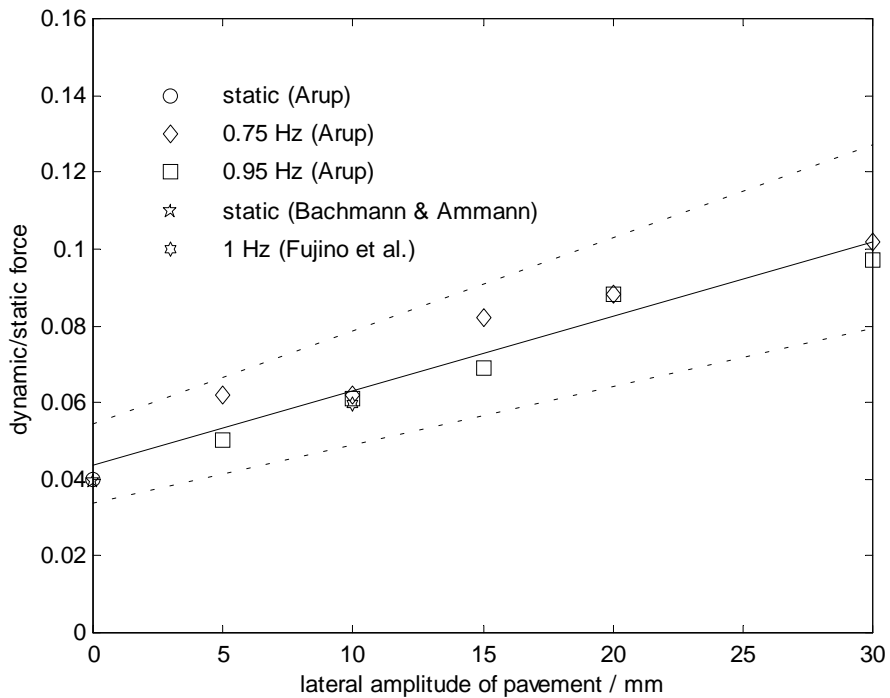


Fig. 2 Measured values of pedestrian lateral dynamic force/static weight as a function of pavement amplitude (after Dallard et al, 2001, fig. 10) with data by Bachmann and Ammann (1987) and Fujino et al (1993) added. The platform in these experiments was 7.3 m long and 0.6 m wide with a handrail along one side. The amplitude of the fundamental component of lateral force is plotted after dividing by the subject's weight. Arup's data is for two different frequencies of pavement oscillation: 0.75 and 0.95 Hz. It appears that subjects walked at a comfortable speed with a walking pace not intentionally "tuned" to the pavement frequency. Fujino's figure is an estimated force amplitude from observations of people walking on a bridge with a 1 Hz lateral mode at an amplitude of about 10 mm. The three added lines (drawn for comparison), are for moving a rigid mass at frequencies of .75 Hz (bottom line), .85 Hz (middle), .95 Hz (top) through an amplitude of 15 mm (at the left) to 35 mm (at the right).

These results were thoroughly investigated following the London Millennium Bridge's problems by its designers, the Arup Partnership, with the results given in Fitzpatrick et al. (2001) and Dallard et al. (2001). Using moving platforms, data was measured on lateral dynamic force and on the probability that a pedestrian would synchronise with pavement lateral vibration. Results obtained by Arup using a

shaking table at Imperial College are shown in fig. 2 which has two other results added. It can be seen that the fundamental component of lateral force increases with platform amplitude but is insensitive to pavement lateral frequency. However walkers were not asked to try to intentionally “tune” their step to the platform’s motion; instead they were asked to walk comfortably for the 7 or 8 paces required to pass over the platform. Fig. 2 has three added lines which demonstrate the ratio of dynamic lateral force/static weight for a rigid mass when oscillated at .75 Hz (bottom line), .85 Hz (middle line) and .95 Hz (top line) when the amplitude of oscillation increases linearly from 15 mm at the left-hand side to 35 mm at the right-hand side. This would apply if a pedestrian were modelled as a rigid mass whose centre-of-mass moved through an amplitude of 15 mm on a stationary pavement and increased linearly with pavement amplitude to 35 mm when the pavement amplitude became 30 mm.

Fujino et al (1993) noticed that a person’s lateral head movement is typically twice that of their feet (laterally) at 1 Hz and  $\pm 10$  mm pavement movement, so it is not surprising that pedestrians do not behave as rigid bodies. But, although they do not act as rigid masses, the lateral force a person generates must react against the inertia of their body, so that the sum of mass $\times$ acceleration for all a person’s component parts must equal the lateral pavement force at all times. Therefore, if an average is calculated for each pedestrian, the results in fig. 2 suggest that their centre-of-mass must be moving about  $\pm 15$  mm when walking comfortably on a stationary pavement, increasing linearly to about  $\pm 35$  mm when the pavement’s lateral movement is  $\pm 30$  mm. The effect of frequency of pavement movement does not seem to have much effect, with measured data for .95 Hz suggesting a slightly lower dynamic/static force ratio than .75 Hz. This is consistent with the natural flexibility of the human frame. Evidently if the pavement were oscillating at a high frequency, the feet and legs would be expected to move, but the upper body would not follow so much and would move relatively less. Movement of the centre-of-mass of the pedestrian would then be significantly different from movement of the pavement.

Let a (scalar) parameter  $\alpha$  relate the amplitude of body movement caused by pavement movement to the amplitude of the pavement. Since, from fig. 3, we deduce that a change in lateral pavement amplitude from 0 to 30 mm causes a change in lateral body movement from 15 to 35 mm,  $\alpha = (35 - 15)/30 = 2/3$ . This data suggests that  $\alpha$  is approximately  $2/3$  at 0.75 Hz and at 0.95 Hz. Because of the complex dynamics of the human frame, it is possible that the effect of different frequencies in this range is small, as this data suggests.

Arup also studied the probability of synchronisation for people using the walking platform at Imperial College and their results are shown in fig. 3. This is the estimated probability that people will synchronise their footfall to the swaying frequency of the platform. The “best-fit” straight line does not pass through the origin. It suggests that people synchronise with each other when there is no pavement

motion but that the probability of synchronisation increases as pavement amplitude increases. Based on this data, the probability of synchronisation is expected to be about 0.4 for small amplitudes up to 10 mm.

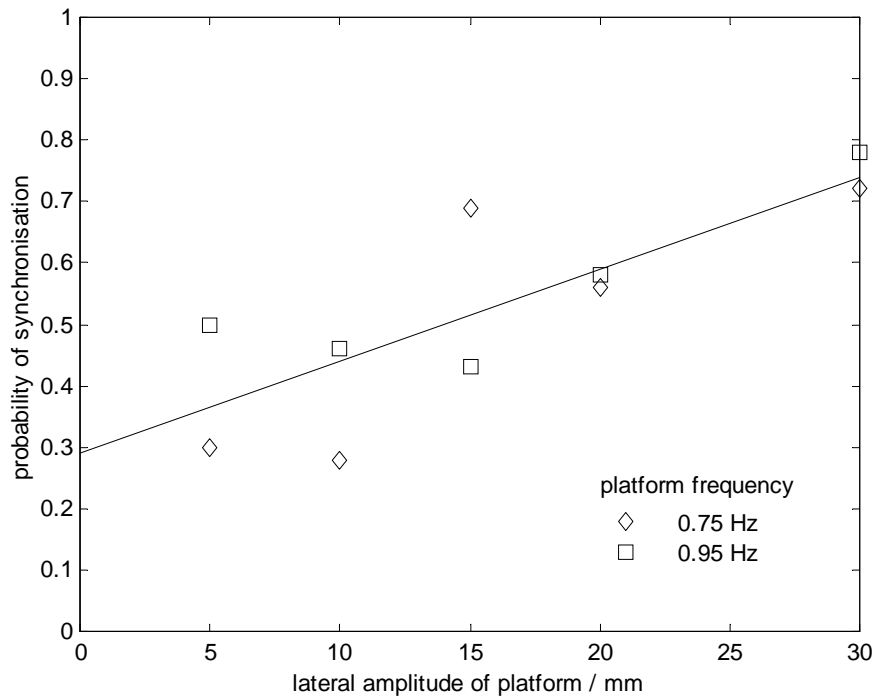


Fig. 3 Probability of synchronisation estimated by Arup from moving platform tests for the same two frequencies of platform lateral oscillation as in fig. 2, 0.75 Hz and 0.95 Hz (after Dallard et al. 2001, fig. 11).

In addition to laboratory tests, Arup conducted a series of crowd tests on the Millennium Bridge. These concluded that pedestrian movement was strongly correlated with lateral movement of the bridge but not apparently with vertical movement. This was attributed in part to the conclusion that pedestrians are “*less stable laterally than vertically, which leads to them being more sensitive to lateral vibration*” (Dallard et al, 1992). However vertical bridge amplitudes were significantly smaller than lateral amplitudes and it was concluded that vertical synchronisation might occur. Vertical vibration control measures were added to the bridge as a precaution against this possibility.

In the following analysis, the assumptions that will be made about pedestrian loading are only appropriate for small-amplitude pavement movements (less than about 10 mm amplitude). For larger amplitudes, people’s natural walking gait is modified as they begin to lose their balance and have to compensate by altering how

they walk. The staggering movement of pedestrians trying to walk on a pavement which has large-amplitude lateral vibration (100 mm amplitude) has been studied by McRobie and Morgenthal (2002a) using a swinging platform. Pedestrian movement was followed by a motion capture system devised by Lasenby (see Gamage and Lasenby, 2002, and Ringer and Lasenby, 2000). The way people walked on a platform moving with such large amplitude varied from person to person. “*A common response was to spread the feet further apart and to walk at the same frequency as the pre-existing oscillations such that feet and deck maintained a constant phase relation.*” although “*Other walking patterns, some involving crossing of the feet, some involving walking in undulating lines were also observed.*” (McRobie and Morgenthal, 2002a). They also found that the lateral forces of the feet-apart gait are phase synchronised to the structure, and approach 300 N amplitude per person, which the authors point out is four times the Eurocode DLM1 value of 70 N for normal walking.

## ANALYSIS OF PEDESTRIAN-BRIDGE INTERACTION

A detailed analysis of the dynamic interaction between walking pedestrians and a flexible bridge will be given in a forthcoming paper (Newland, 2003b). However simplified calculations can be made to determine the limiting value of structural damping required to ensure stability.

Consider the interaction between pedestrians of effective modal mass  $m$  walking on a bridge with a vibration mode of (modal) mass  $M$  and stiffness  $K$ , for small-amplitude vibrations. The interaction (modal) force which is transmitted from the pedestrians to the bridge, and vice versa, is  $f$ . This system is shown in fig. 4 where  $z(t)$  is the (effective) modal displacement of the pedestrians’ centre-of-mass and  $y(t)$  measures the modal displacement of the bridge’s pavement or walkway.

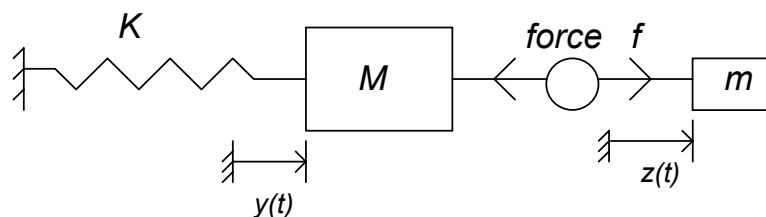


Fig. 4 Interaction between a bridge mode with modal mass  $M$  and stiffness  $K$  and pedestrians with modal mass  $m$ . The (modal) force transmitted between the pavement and pedestrians is  $f$ . The symbol O recognises that there is a complex interaction between pedestrians and bridge recognised by the time delay  $\Delta$  and the correction factors  $\alpha$  (in equation 4) and  $\beta$  (in equation 12).



The equations of motion are:

$$M \ddot{y}(t) + K \dot{y}(t) = -f(t) \quad (1)$$

$$f(t) = m\ddot{z}(t) . \quad (2)$$

Structural damping is not included, but if we assume viscous damping with coefficient  $C$ , then, combining (1) and (2) gives

$$M \ddot{y}(t) + C \dot{y}(t) + K y(t) + m\ddot{z}(t) = 0. \quad (3)$$

Because people respond to movement of the pavement, we assume that their amplitude is given by  $\alpha y$  (generally less than the amplitude of the pavement, and therefore of their feet,  $y$ ) and with a time difference  $\Delta$  so that

$$z(t) = \alpha y(t - \Delta) . \quad (4)$$

If steady-state harmonic conditions occur, then a solution is

$$y(t) = Y \exp(i\omega t) \quad (5)$$

and, on substituting (4) and (5) into (3) and defining the phase angle

$$\phi = \omega \Delta , \quad (6)$$

we find two equations, one each for the real and imaginary parts of (3), both of which must be zero, so that

$$-M\omega^2 + K - \alpha m\omega^2 \cos\phi = 0 \quad (7)$$

$$C\omega + \alpha m\omega^2 \sin\phi = 0 . \quad (8)$$

These give the limiting condition for stable simple harmonic motion. The structural damping coefficient  $C$  must exceed the value given in (8) if motion is to remain stable, and so the condition for stability is

$$C > -\alpha m\omega \sin\phi . \quad (9)$$

In this expression, the phase angle  $\phi$  depends on how the walking pedestrians synchronise their step with movement of the pavement and the “worst case” will occur when  $\phi = -\pi/2$ , when  $C$  will be greatest. Then  $\cos\phi = 0$  in (7), and so the frequency is the undamped natural frequency  $\omega = \sqrt{K/M}$ . In terms of the structural damping ratio  $\zeta$  for the bridge with no pedestrians, defined by

$$\zeta = C / 2\sqrt{KM}, \quad (10)$$

the condition for stability can be written finally as

$$2\zeta > \alpha m / M. \quad (11)$$

This says that the structural damping ratio required for stability has to exceed a limiting value given by the modal mass of the pedestrians divided by the modal mass of the bridge, multiplied by a factor  $\alpha$  which relates the amplitude of body movement of pedestrians to the amplitude of the pavement.

A complication is that not everybody naturally falls into step as small amplitude movement of the pavement occurs and only a proportion  $\beta$  may be considered to have done so, reducing the effective pedestrian mass from  $m$  to  $\beta m$ , so that the final stability criterion is

$$2\zeta > \alpha \beta m / M \quad (12)$$

This result gives the key to calculating the amount of structural damping needed to prevent self-excited oscillations building up. Using values for the parameters  $\alpha$  and  $\beta$  derived from the experimental data described above, it is possible to quantify how much damping will be needed in any particular circumstance. Alternatively, if the structural damping ratio  $\zeta$  for a given mode is known, the safe modal mass of pedestrians for that mode can be calculated from (12).

## PEDESTRIAN SCRUTON NUMBER

McRobie has pointed out that there is an analogy between the wind excitation of flexible structures and people excitation of bridges (McRobie and Morgenthal, 2002b). The tendency for vortex shedding to cause wind-excited structural oscillations is measured by the non-dimensional Scruton Number which is a product of damping and the ratio of representative structural and fluid masses. The usual definition is

$$S_c = 4\pi\zeta M / \rho b^2 \quad (13)$$

where  $\zeta$  is the damping ratio of the relevant mode,  $\rho$  is air density and, for a cylindrical structure of diameter  $b$ ,  $M$  is the mass per unit length of the structure. Large Scruton numbers are preferable. McRobie suggested that the same approach should be taken for pedestrian-excited vibration, distinguishing between vertical and lateral vibration to allow for the different human responses to vertical and lateral

pavement movement. The definition of pedestrian Scruton number is arbitrary, but for the purpose of this paper

$$S_{cp} = 2\zeta M / m \quad (14)$$

where

$S_{cp}$	=	Pedestrian Scruton number
$\zeta$	=	modal damping ratio
$M$	=	modal mass, or, for a uniform deck, bridge mass per unit length
$m$	=	modal mass of pedestrians, or, for a uniform bridge deck with evenly spaced pedestrians, pedestrian mass per unit length

For this definition, and substituting from (12), we conclude that, for stability, it is necessary that

$$S_{cp} > \alpha \beta. \quad (15)$$

The analysis uses the model in fig. 4. As defined already,  $\alpha$  is the ratio of movement of a person's centre-of-mass to movement of the pavement, which from the results given previously has been found to be about 2/3 for lateral vibration in the frequency range 0.75 to 0.95 Hz, and  $\beta$  is the correlation factor for individual people synchronising with pavement movement, which is typically about 0.4 for lateral pavement amplitudes less than 10 mm.

Typical data has been assembled from the sources available (which are somewhat meagre and generally incomplete) and is reproduced below, fig. 6 (for lateral vibration) and fig. 7 (for vertical vibration). It can be seen that the pedestrian Scruton numbers for typical modes of the London Millennium Bridge were initially very low, less than the limit given by (15) for the estimated numerical values of the experimental parameters  $\alpha = 2/3$  (ratio of body movement to pavement movement) and  $\beta = 0.4$  (the correlation factor for people synchronising with pavement movement). After modification to artificially increase the bridge's damping, the corresponding Scruton numbers are much higher, well above the upper limit from (15) (with  $\alpha = \beta = 1$ ) and well above an alternative limit (17) suggested by Arup (see below).

For lateral vibration, fig. 6, the estimated pedestrian Scruton numbers for both Fujino's bridge in Japan and the Auckland Harbour Bridge lie below the lower limit from (15). In the case of vertical vibration, fig. 7, there is additional data available in McRobie and Morgenthal (2002) for a number of "lively" bridges, all of which falls below the lower limit. The Auckland Harbour Bridge is interesting because it falls between the upper and lower limits calculated by using (15). This data was measured during the course of a marathon race in 1992 when a large number of runners crossed one of the two-lane roadways. It is recorded that vertical amplitudes of up to 3 mm

were experienced in a frequency range of 2.6 to 3 Hz, which is noticeable by runners. From this it may be concluded that vertical bridge oscillation of serious amplitudes could be excited by the natural synchronisation of a large enough crowd of runners (as distinct from a marching army in the traditional sense). That is why the decision was taken to artificially increase the modal damping of vertical as well as lateral modes for the London Millennium bridge.

### ARUP'S ANALYSIS

As a result of a series of crowd tests, and an energy analysis of vibrational power flow, Arup concluded that the correlated lateral force per person is related to the local velocity by an approximately linear relationship which was found to hold for lateral frequencies in the range 0.5 to 1 Hz (pacing frequency 1 to 2 Hz). It is interesting that, within this frequency range, the results again appear to be insensitive to frequency. If  $k$  is defined as the slope of a graph of the amplitude of average lateral force per person plotted against the amplitude of pavement lateral velocity, Arup found that  $k \approx 300$  Ns/m for a bridge mode in the frequency range 0.5 to 1 Hz.

By assuming that each person generates a velocity-dependent force which acts as negative damping, and making a modal calculation, they also concluded that vibrational energy in the mode would not increase if (Dallard et al, 2001, p. 28, equation 9)

$$\zeta > Nk / (8\pi f M) \quad (16)$$

where  $\zeta$  is the modal damping ratio,  $f$  is the natural frequency, and  $M$  is the modal mass. For this condition, the positive modal damping exceeds the negative damping generated by pedestrian movement. On substituting (16) into (14) to calculate the required pedestrian Scruton number, we find that the limiting condition is

$$S_{cp} > k / (2\pi f m_o) \quad (17)$$

where  $m_o$  is the mass per person for whom  $k=300$  Ns/m in the frequency range 0.5 to 1.0 Hz. This result has been added to fig. 6 using  $m_o = 75$  kg per person.

### CONCLUSIONS

The analysis in this paper shows that bridge vibration will become unstable when the live load, represented by people of mass  $m$  per unit length, is too great a proportion of the bridge's dead load stemming from its mass  $M$  per unit length.

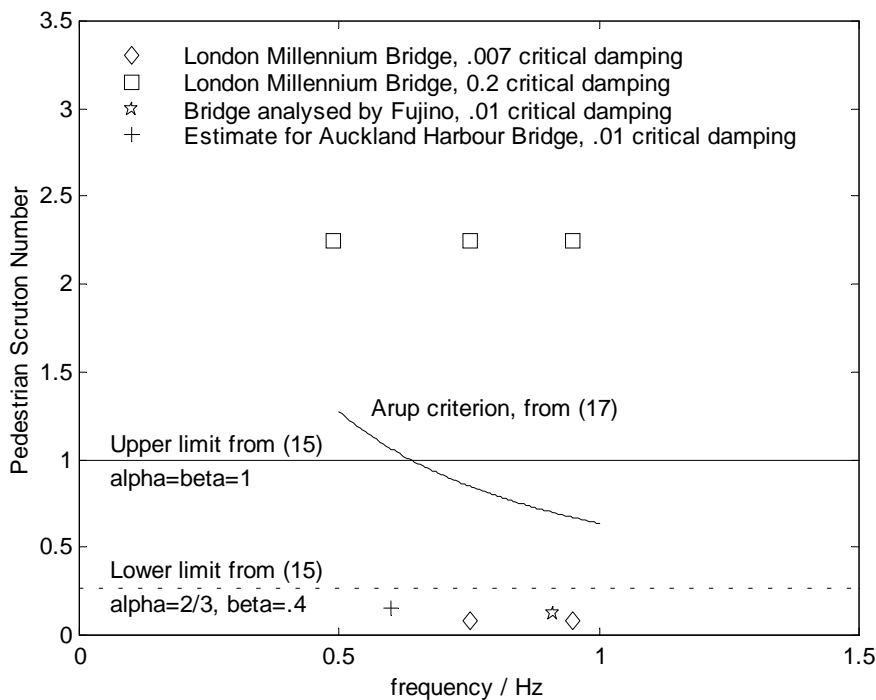


Fig. 6 Some collected data on Pedestrian Scruton Number for lateral modes. Limiting values from equation (15) with  $\alpha = 2/3, \beta = 0.4$  and with  $\alpha = \beta = 1$ , and from equation (17), are shown for comparison.

The permissible  $m/M$  ratio depends on the amount of damping present in the appropriate vibrational modes that will be excited by the pacing rate of pedestrians because problems only arise when their excitation frequency is close to a natural frequency of a lightly-damped vibrational mode. Specifically according to (12) it is necessary for stability that,

$$2\zeta > \alpha \beta m/M \quad (18)$$

where  $\zeta$  is the ratio of critical damping in the mode and  $\alpha$  and  $\beta$  are experimentally-determined factors. From the data so far available, it appears satisfactory to assume that  $\alpha=2/3$  relates movement of a person's centre-of-mass to movement of their feet (from the slope in fig. 2) and  $\beta=0.4$  for bridge amplitudes up to about 10 mm (from fig. 3). However these numbers derive from a limited number of experiments on lateral vibration, and can only be regarded as provisional for lateral vibration and a first indication of possible numbers for vertical vibration for which measurements have not yet been made.

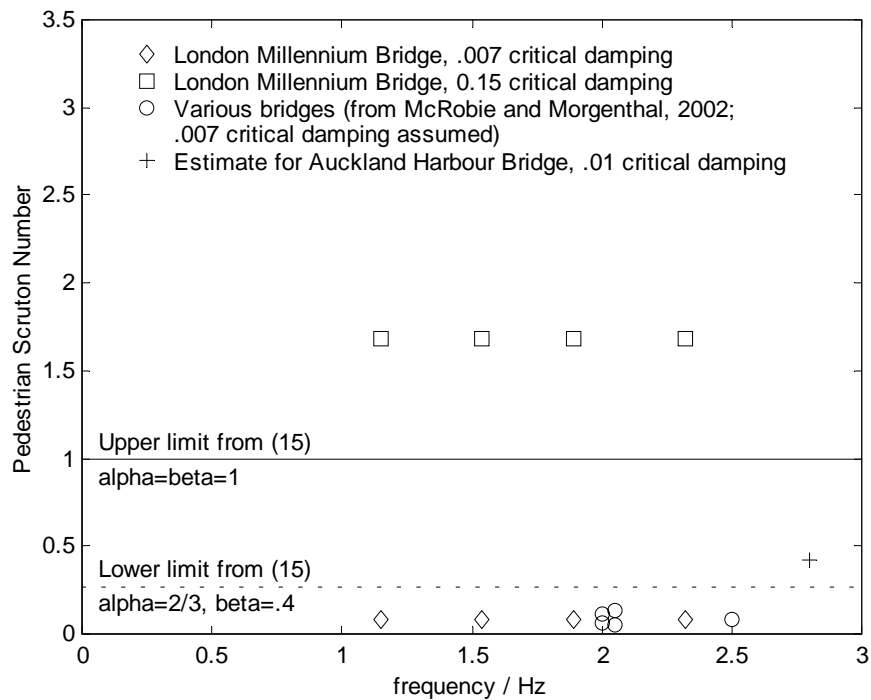


Fig. 7 Some collected data on Pedestrian Scruton Number for vertical modes.

The dependence of a damping stability criterion on a mass ratio is consistent with the experience of vortex-excited oscillations in aeroelasticity and application of the pedestrian Scruton number defined by (14) allows this criterion to be expressed alternatively by (15). The results plotted in figs. 6 and 7 show clearly that troublesome bridges all have pedestrian Scruton numbers that fall below the lower limit given by (15). Similarly the London Millennium Bridge after modification lies well above the upper limit on pedestrian Scruton number drawn by assuming that both the factors  $\alpha$  and  $\beta$  in (15) are unity, which is likely to be a very pessimistic assumption.

The collection of more experimental data will be helpful in verifying these conclusions but, in the meantime, the provision of damping above the upper limit derived from (15) is likely to provide reasonable assurance that unexpected pedestrian-excited bridge oscillations will not occur. The introduction of damping by a combination of frame-mounted viscous dampers and tuned-mass vibration absorbers, which cured the London Millennium Bridge's vibration problem, met this criterion.

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